

```
In[370]:=
SetDirectory[NotebookDirectory[]];
<< MaTeX`
texStyle = {};
SetOptions[MaTeX,
  "BasePreamble" → {"\\usepackage{amsmath}", "\\usepackage{xcolor}",
    "\\usepackage{fourier}", "\\usepackage{ebgaramond}"}, FontSize → 11];
```

```
In[374]:=
frame[legend_] :=
  Framed[legend, FrameStyle → Thin, RoundingRadius → 10, FrameMargins → 0];
```

```
In[375]:=
αλ[λ_, μ_] := μ / (λ + 2 μ);
λα[α_, μ_] := μ (1 / α - 2);
CWeyl[d_, λ_, μ_] :=
  ((λ + 2 μ) ^ (-d / 2) + (d - 1) μ ^ (-d / 2)) / (4 Pi) ^ (d / 2) / Gamma[1 + d / 2];
BDirLiu[d_, λ_, μ_] :=
  - ((d - 1) / μ ^ ((d - 1) / 2) + 1 / (λ + 2 μ) ^ ((d - 1) / 2)) / 4 / (4 Pi) ^ ((d - 1) / 2) /
  Gamma[1 + (d - 1) / 2];
BDir[d_, α_, μ_] := -μ ^ ((1 - d) / 2) / (2 ^ (d + 1) Pi ^ ((d - 1) / 2) Gamma[(1 + d) / 2])
  (4 (d - 1) / Pi NIntegrate[τ ^ (d - 2) ArcTan[Sqrt[(1 - α τ ^ (-2)) (τ ^ (-2) - 1)]],
    {τ, Sqrt[α], 1}] + α ^ ((d - 1) / 2) + d - 1);
BDirSaVa2[α_, μ_] := 1 / (4 Pi Sqrt[μ]) (-1 - Sqrt[α] -
  4 / Pi NIntegrate[ArcTan[Sqrt[(1 - α / ξ ^ 2) (1 / ξ ^ 2 - 1)]], {ξ, Sqrt[α], 1}]);
BDirSaVa3[λ_, μ_] := -1 / (16 Pi) (3 λ ^ 2 + 13 λ μ + 16 μ ^ 2) / (λ ^ 2 μ + 5 λ μ ^ 2 + 6 μ ^ 3);
γR[α_] := Sqrt[Min[x /. Solve[x^3 - 8 x^2 + 16 (-1 + α) x == 0, x, Reals]]];
Bfree[d_, α_, μ_] := μ ^ ((1 - d) / 2) / (2 ^ (d + 1) Pi ^ ((d - 1) / 2) Gamma[(1 + d) / 2])
  (4 (d - 1) / Pi NIntegrate[
    τ ^ (d - 2) ArcTan[(τ ^ (-2) - 2) ^ 2 / (4 Sqrt[(1 - α τ ^ (-2)) (τ ^ (-2) - 1)])],
    {τ, Sqrt[α], 1}] + α ^ ((d - 1) / 2) + d - 5 + 4 γR[α] ^ (1 - d));
BfreeSaVa2[α_, μ_] :=
  1 / (4 Pi Sqrt[μ]) (4 / γR[α] - 3 + Sqrt[α] + 4 / Pi NIntegrate[ArcTan[
    (2 - 1 / ξ ^ 2) ^ 2 / (4 Sqrt[(1 - α / ξ ^ 2) (1 / ξ ^ 2 - 1)])], {ξ, Sqrt[α], 1}]);
BfreeSaVa3[λ_, μ_] :=
  1 / (16 Pi) (3 (λ + 2 μ) ^ 2 - 3 (λ + 2 μ) μ + 2 μ ^ 2) / ((λ + 2 μ) μ (λ + μ));
```

```
In[386]:=
MyNlambdaSimple[evs_, Λ_] := Sum[UnitStep[Λ - evs[[j]]], {j, 1, Length[evs]}];
```

```
In[387]:=
str1 = "24.47452712    24.47452713    50.29490584    57.3925945    64.8938698
 72.04546225    92.75173509    92.75173871    106.7443192    106.7443255
134.4118521    142.7120076    142.7120097    154.4167131    156.7771305
166.3961097    175.357488    187.8052354    198.9745325    198.9745793
229.0737067    229.0737125    243.9402376    245.6251193    254.826607
254.8267257    262.1540404    279.3148547    282.4012701    282.401328
296.151469    305.4441348    318.8618392    347.5218045    347.5219511
353.1707538    363.7581029    364.5079603    373.7818815    373.7819444
```

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394.7542079      394.7544849      404.9963395      411.8109544      412.4952853
412.4955202      432.8375439      445.9325415      471.2917673      471.2919693
474.2537769      493.1231478      493.1257056      499.3752876      513.7605595
517.9713671      519.0496053      538.5912459      538.5913404      551.2729437
555.133619      557.7649707      557.7656832      571.403361      598.7562027
598.7569927      612.1549029      628.8146406      631.4077446      631.4081803
639.6876776      641.8167802      647.649411      656.3895733      671.4828257
671.485693      701.2478231      710.5777728      710.5798659      718.1698601
719.2770693      735.7064184      740.3246996      740.3271285      757.3991393
770.5579469      770.5585241      784.6424704      791.5418217      791.5419802
798.15031      810.0171053      810.0252218      838.9055685      840.4335127
841.5531533      849.6291285      849.631283      864.8244044      891.7678029
891.7691917      897.2836236      899.6109161      901.8264362      901.8276384
911.9727268      952.9461149      957.2096456      971.9157367      974.8543634
976.454967      988.5919425      988.5939506      997.6458097      998.1934998
1011.49532      1011.498882      1014.458449      1029.103006      1029.115418
1045.69384      1045.699017      1046.403812      1068.210644      1076.198996
1076.205915      1084.6896      1085.033372      1096.400499      1142.962692
1142.969847      1149.496791      1159.945646      1165.303804      1165.31346
1191.796222      1204.499635      1204.528275      1209.360048      1209.364632
1213.889686      1215.531158      1220.918697      1220.918771      1237.797541
1238.202523      1238.555949      1251.421063      1269.958374      1280.698661
1292.197595      1292.206404      1297.847528      1305.332513      1305.340023
1337.123626      1337.126592      1339.668641      1358.703508      1366.925042
1366.929837      1378.450256      1395.111163      1399.310921      1409.836581
1409.839989      1431.862551      1432.870994      1444.382458      1463.331068
1463.344209      1466.368689      1466.408734      1485.824047      1492.311342
1494.652439      1504.955451      1505.279406      1505.287799      1522.097776
1525.478634      1527.413044      1536.242625      1536.252304      1561.817085
1570.469404      1570.492579      1577.298231      1581.483988      1608.009767
1611.587244      1611.62051      1614.215925      1637.096711      1651.030794
1665.156604      1665.171137      1679.456819      1679.502234      1695.094251";
str2 = "51.70921989      52.18171182      52.1817157      102.2050755      103.7480975
103.7481037      117.4264911      166.0571944      172.5786762      172.5787291
184.1566449      201.1545519      203.1175164      234.6344209      243.3240676
243.3242455      265.2174518      283.8444891      283.8447495      298.63723
308.9242335      308.9242814      355.6290305      367.6001726      389.2180201
395.5193646      400.196495      401.2049412      401.2053987      403.4954254
450.0113048      450.0114253      483.8857785      483.8882029      497.1896802
497.1954728      518.2867645      533.4075955      533.784189      539.6462747
539.6468872      557.845514      583.9195941      593.9511302      601.1812062
639.7645019      639.7666529      643.5265048      645.5819119      690.7701256
700.1295169      700.1317754      713.4910891      713.4932903      721.9411791
721.9425097      737.8992202      793.7622479      796.6958712      806.8488343
806.8715817      811.3252231      823.2747622      830.0464842      838.0090568
838.7588215      891.2843103      891.2857262      895.6178973      896.4518059
906.4874247      906.4948607      912.4106136      954.5869239      954.5907866

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982.2445498 982.2553074 995.6308106 998.0710164 1032.345617
1032.368683 1062.682749 1062.982742 1064.56933 1093.165475
1094.988997 1094.996321 1104.957955 1115.110474 1127.893823
1137.980585 1142.31059 1206.102655 1206.182279 1227.040089
1227.056657 1241.186212 1242.723802 1244.641765 1244.653581
1250.965355 1250.973443 1283.221769 1283.240992 1313.816345
1334.230736 1334.242975 1350.613478 1352.115201 1352.122035
1354.726932 1371.503632 1406.105915 1418.400759 1423.97428
1437.792498 1439.128694 1457.322898 1457.389356 1462.737888
1470.78282 1471.724299 1503.752558 1560.409936 1560.422359
1573.586198 1573.618997 1584.407465 1588.580698 1592.140765
1592.189545 1605.467653 1620.705648 1620.773586 1654.671907
1654.71662 1681.46973 1681.617156 1700.391025 1707.466881
1753.736531 1763.255769 1763.274985 1766.618525 1770.711989
1789.669281 1791.131074 1791.18824 1794.831731 1822.1192
1822.94294 1836.26012 1847.41904 1847.447739 1889.130949
1897.085059 1899.967058 1916.694048 1916.719698 1946.463492
1947.008928 1952.048049 1952.102423 1979.842654 1980.022557
1989.768862 1989.840767 2023.417318 2023.517358 2037.017668
2037.73307 2056.212064 2056.244059 2116.227713 2122.367694
2128.420654 2128.454663 2134.653811 2153.27443 2164.454671
2165.060879 2182.29174 2182.37586 2214.189683 2217.767395
2225.359428 2234.921592 2235.507182 2272.60244 2275.339488
2276.817577 2279.08259 2304.43956 2304.597472 2322.850445
2364.589509 2364.613969 2366.160063 2398.353901 2398.451306";
str3 = "52.3134236 91.47926782 91.47937066 127.9118106 153.045409
166.9946786 189.1410276 189.1433564 243.8482642 243.8509262
245.9489886 268.3186962 278.4927218 326.1510139 326.1576692
346.270171 362.6774739 379.3298702 379.3396619 403.4468766
424.7602543 434.272121 474.7143434 474.7630296 499.2594766
503.4210141 503.455139 510.8214267 553.7466664 553.794035
600.5058649 618.0615865 621.2654701 630.0083253 638.7159763
651.6430951 651.6470775 693.6846911 704.2878463 720.3127763
720.3496146 767.6188354 767.7616412 773.4941525 773.6527388
811.2175171 823.0486025 838.8528668 857.805508 864.203792
864.271111 865.9712593 928.4118595 938.2223671 965.7748833
976.9568721 977.0716242 993.9976833 1003.964437 1004.022818
1004.237327 1004.375683 1025.329577 1025.598114 1044.73618
1055.847234 1115.274649 1117.817282 1118.055274 1134.63921
1142.002312 1202.931214 1203.678338 1212.971682 1217.653212
1233.898905 1234.594398 1255.98796 1256.037566 1274.6453
1274.818691 1318.35485 1320.788047 1321.236717 1328.53537
1381.996554 1411.767285 1412.360489 1429.2643 1432.389971
1437.635856 1438.055558 1451.868688 1457.860721 1476.837779
1490.476089 1519.661147 1528.526013 1548.771986 1549.149272
1611.526212 1611.909078 1633.032672 1642.479425 1657.095152
1658.105141 1674.099824 1685.940431 1687.479306 1690.073958

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1728.657585 1728.918957 1748.650734 1749.865833 1790.027888
1790.234521 1798.558296 1809.702894 1816.142641 1818.761061
1876.492109 1882.731972 1883.670124 1884.912941 1945.084351
1949.14023 1950.389268 1988.790382 1989.826185 1990.039928
1990.473747 1997.97159 2003.90017 2034.224332 2035.605137
2062.40672 2062.789259 2124.632849 2126.266352 2128.840704
2144.926981 2154.861747 2188.9266 2208.705725 2215.122496
2224.28033 2234.96253 2238.983924 2242.253051 2257.863047
2259.519202 2274.210398 2283.036845 2284.272512 2285.524832
2344.394825 2356.753561 2386.458228 2395.115976 2408.646419
2409.051612 2436.615896 2437.781579 2453.746381 2456.107729
2540.046351 2540.331381 2541.369373 2542.191634 2544.704193
2546.096471 2551.04421 2584.720336 2599.064477 2600.638146
2629.677056 2649.354875 2654.413772 2657.40287 2657.932397
2676.690151 2679.883648 2713.888751 2721.563661 2740.647114
2753.575955 2825.156836 2833.347529 2851.31676 2852.141391
2854.387654 2868.392399 2870.494226 2871.398503 2872.266356
2876.468155 2913.014966 2915.369728 2921.609201 2924.790263";

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In[390]:=

```

evsSquareβ05 = Flatten@ToExpression@StringSplit[str1];
evsSquareβ4 = Flatten@ToExpression@StringSplit[str2];
evsSquareβ101 = Flatten@ToExpression@StringSplit[str3];
bthis1 = BDir[2, 2 / 3, 1];
bthis2 = BDir[2, 1 / 5, 1];
bthis3 = BDir[2, 1 / 102, 1];

```

In[396]:=

```

xtsq1 = Table[{x, MaTeX[x]}, {x, {800, 1600}}];
xtsq2 = Table[{x, MaTeX[x]}, {x, {1200, 2400}}];
xtsq3 = Table[{x, MaTeX[x]}, {x, {1400, 2800}}];
ytsq = Table[{y, MaTeX[y]}, {y, -40, -10, 10}];

```

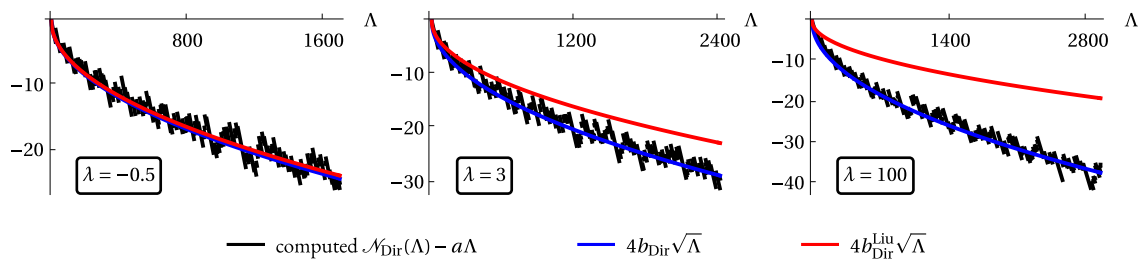
In[400]:=

```

figsq1 = Plot[{MyNlambdaSimple[evsSquareβ05, Δ] - CWeyl[2, -0.5, 1] Δ,
  4 bthis1 Sqrt[Δ], 4 BDirLiu[2, -0.5, 1] Sqrt[Δ]}, {Δ, 0, 1696},
  PlotStyle → {Black, Blue, Red}, Ticks → {xtsq1, ytsq},
  AxesLabel → {MaTeX["\\Lambda"], None}, Epilog → Inset[
    Framed[MaTeX["\\lambda=-0.5"], RoundingRadius → 2, ContentPadding → False,
      FrameMargins → Tiny], Scaled[{0.1, 0}], Scaled[{0, 0}]]];
figsq2 =
  Plot[{MyNlambdaSimple[evsSquareβ4, Δ] - CWeyl[2, 3, 1] Δ, 4 bthis2 Sqrt[Δ],
    4 BDirLiu[2, 3, 1] Sqrt[Δ]}, {Δ, 0, 2400}, PlotStyle → {Black, Blue, Red},
  Ticks → {xtsq2, ytsq}, AxesLabel → {MaTeX["\\Lambda"], None}, Epilog →
  Inset[Framed[MaTeX["\\lambda=3"], RoundingRadius → 2, ContentPadding →
    False, FrameMargins → Tiny], Scaled[{0.1, 0}], Scaled[{0, 0}]]];
figsq3 = Plot[{MyNlambdaSimple[evsSquareβ101, Δ] - CWeyl[2, 100, 1] Δ,
  4 bthis3 Sqrt[Δ], 4 BDirLiu[2, 100, 1] Sqrt[Δ]}, {Δ, 0, 2925},
  PlotStyle → {Black, Blue, Red}, Ticks → {xtsq3, ytsq},
  AxesLabel → {MaTeX["\\Lambda"], None}, Epilog → Inset[
    Framed[MaTeX["\\lambda=100"], RoundingRadius → 2, ContentPadding → False,
      FrameMargins → Tiny], Scaled[{0.1, 0}], Scaled[{0, 0}]]];
legendsq = LineLegend[{Black, Blue, Red}, MaTeX[{"\\text{computed}
  }\\mathcal{N}_{\\mathrm{Dir}}(\\Lambda) - a\\Lambda\\quad\\quad",
  "4b_{\\mathrm{Dir}}\\sqrt{\\Lambda}\\quad\\quad",
  "4b_{\\mathrm{Dir}}^{\\mathrm{Liu}}\\sqrt{\\Lambda}"}],
  LegendFunction → None, LegendLayout → {"Row", 1}];
figsq = GraphicsColumn[{GraphicsRow[{figsq1, figsq2, figsq3},
  ImageSize → Full, Spacings → {Scaled[0.05], Automatic}],
  legendsq}, ImageSize → Full, Spacings → {0, Automatic}]

```

Out[404]:=



In[\*]:= Export["figsq.pdf", figsq]

Out[\*]=

figsq.pdf

In[405]:=

```

strfr1 = "0      0      0      9.426909466      9.426909585
 10.43266371   13.06216344   14.03415763   19.73921027   29.08345184
 29.08345215   32.94601647   44.60423164   50.83942515   50.83942763
 52.77546069   60.66480941   60.66481025   67.76480679   78.95692801
 79.30605527   79.30607387   84.3184671    103.1082695   103.1083365

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107.9439506    115.4073194    122.4281302    132.5721029    132.7950436
134.2740912    138.6930148    138.6930669    152.3829951    152.38312
161.0447604    170.3470573    177.6539277    183.6077879    183.6077995
196.3823903    196.3825327    197.0457187    225.8279778    225.8285726
233.7726645    234.8969837    241.1462012    241.4239619    241.4242618
250.4508001    253.1556278    260.5511762    260.5514249    274.3000919
283.5688561    294.7901796    296.0056828    296.0061105    299.9822202
315.8333182    321.3238243    341.8697076    341.8697872    352.9493818
352.9497315    367.3581938    367.3591131    368.7250227    369.6092565
370.0400678    372.068083     380.0477424    393.3449679    393.3463473
411.6828462    411.6831965    412.4708299    426.1221431    426.1245012
438.6164962    447.6579235    463.4517794    463.9995738    470.3589278
489.8694297    489.870456     492.1213748    493.5027132    493.5828218
512.2280662    512.2297065    520.4261119    527.0115786    529.7588167
529.7592889    535.9522808    557.144114     558.2419079    562.986677
562.9876994    568.1829681    583.9715145    583.9746513    598.6188033
598.6240582    605.4552416    621.5187421    621.5195996    636.8014442
642.1933001    642.1961103    650.2415382    650.3860264    657.1596774
657.1749403    686.4546442    695.9972162    701.7516708    704.5911054
705.8276751    705.8364384    709.3338813    710.6770739    733.0245954
738.2621494    741.4519029    741.4540489    753.3284369    753.335935
754.5322838    764.2825736    792.3386207    803.5765863    806.8842458
808.6059947    808.6111237    810.6384273    810.9641212    826.6434216
833.3092641    833.3105895    839.2671779    849.5475881    849.550866
866.4206598    866.4368322    881.0651665    881.066098     884.2213117
911.4551176    915.9579429    930.3758852    930.3896864    937.6713179
946.1436902    952.5068109    952.5122291    967.1917595    967.3881625
980.7818202    982.7404633    982.7740704    985.7390283    988.5937408
998.1148254    998.7395638    998.755951     1012.197828    1012.209836
1032.072008    1032.098237    1038.939062    1045.515581    1062.191283
1066.339069    1088.53878     1088.558162    1089.492566    1096.025971
1097.504938    1105.430332    1114.272804    1114.277254    1130.046249
1130.053907    1171.708207    1171.78116     1174.886749    1182.66895
1182.689452    1186.645408    1187.005847    1196.429011    1198.277887
1206.741921    1209.65151     1221.202714    1221.208738    1225.778511
1242.652085    1247.58837     1252.249329    1252.272014    1263.6725
1277.813242    1277.864048    1288.729965    1290.298346    1292.06607
1294.655524    1294.689343    1297.750264    1314.657601    1329.643618
1330.118934    1330.124571    1334.37201     1376.169162    1376.214127
1401.397437    1401.439005    1401.552836    1409.606604    1418.642651
1428.199491    1431.41784     1431.440955    1436.683862    1436.720765
1449.200945    1449.20861     1461.125805    1479.562139    1481.344632
1483.225333    1491.691613    1491.704841    1491.821576    1507.157627
1508.550416    1515.702066    1517.69949     1517.723209    1535.395582
1535.471297    1544.145027    1544.211573    1565.900667    1565.936035";
strfr2 = "0      0      0      15.60446171    19.73921097
20.33060132    20.33060201    30.8010126     41.09119232    41.09119472

```

56.99124891	58.27092571	77.54425258	77.54426798	78.95696549
84.57771501	92.61666964	113.9815665	115.4988094	115.4988321
137.8525495	137.8526152	145.2080034	157.0296074	170.3689479
177.6543763	186.5811943	186.5812287	186.6565184	205.0589956
205.0590294	237.9407348	238.4156188	238.4156571	255.1436725
255.1437599	268.937713	271.3078375	272.9918356	292.7806793
295.7342593	306.0379775	306.0381754	315.8357307	347.9892432
366.5753664	366.5759552	374.6643925	374.6664989	379.2686276
381.1807516	396.0272616	396.0277837	425.4923396	425.9031521
425.9050921	444.8486132	451.5917885	480.6557674	488.2886499
493.5122199	493.7701128	494.3233831	500.6576625	518.3995035
527.2247308	534.3814682	534.3839644	541.9231273	541.9243134
576.1957772	576.1981155	597.2335595	597.2367037	600.4366852
626.5991003	626.6045717	637.5682866	663.5395618	663.540888
669.0241055	671.0688899	679.4563231	703.8932451	704.3729546
709.9303037	709.9337262	710.7033139	712.3469174	753.3952189
778.1062073	778.1079181	805.8932666	810.1275454	812.756395
814.5351615	814.5442887	832.8240914	832.8320499	835.1220348
835.1287932	852.3336475	865.6433502	865.649718	886.8719553
886.8829011	897.8936224	918.571991	920.1152571	935.859422
964.591566	967.4571006	967.7368183	970.0006169	978.2225849
987.215356	1000.349143	1000.390479	1034.281277	1035.752434
1035.800289	1043.280001	1072.102687	1072.111211	1076.787274
1092.273581	1092.27779	1110.409529	1110.422836	1116.899449
1116.914903	1117.279252	1170.638311	1208.239511	1208.262277
1210.093342	1214.82406	1216.441919	1227.530101	1230.000138
1230.015133	1246.948671	1254.524688	1260.514132	1263.819317
1265.282938	1276.075234	1298.375032	1298.824612	1308.479636
1308.495565	1329.210918	1339.328362	1360.521866	1360.527707
1393.179386	1402.213687	1402.251765	1426.550159	1426.562679
1427.928487	1427.957756	1436.092631	1436.181578	1458.626322
1458.685707	1465.376718	1466.689331	1492.245802	1499.720758
1499.74774	1559.649411	1559.681888	1572.318134	1573.314615
1586.022346	1590.441919	1592.991324	1594.357622	1599.936849
1603.680973	1616.766894	1618.562996	1623.794137	1679.868408
1680.394782	1680.836384	1680.968119	1682.452064	1726.195459
1726.299748	1753.289263	1758.115495	1758.161036	1781.560785
1783.448377	1783.466476	1785.761484	1785.786065	1795.623861
1795.707809	1806.862034	1824.116008	1836.657732	1836.706819
1848.220714	1848.261285	1864.26118	1920.24606	1923.551687
1939.057009	1939.167011	1952.970552	1953.181616	1964.648057
1975.868612	1977.80985	1978.404693	1981.395933	1985.139356
1990.273182	1990.478345	1993.825405	1993.877979	2003.093103
2013.532876	2055.190326	2090.338192	2108.497001	2114.250469
2114.309844	2119.182448	2138.502284	2138.547958	2175.313967
2179.34797	2179.383998	2180.49281	2180.51758	2187.441924
2187.563311	2192.002008	2192.035234	2237.880134	2239.136395

```

2239.161116    2240.832152    2276.01088    2276.192338    2282.873238";
strfr3 = "0    0    0    15.77472163    19.7392192
23.1190593    23.11906806    36.67811371    42.14207716    42.14212471
62.79639989    69.47293846    78.95732576    84.64519802    84.64548229
87.25646389    118.8252651    118.8257258    122.9598543    142.9947082
142.9959902    153.7326011    167.3297603    168.9587306    177.65898
197.6369549    200.8332164    200.8343716    232.5379    232.5384164
241.2496976    243.5595895    243.5608996    281.1232304    288.5814086
304.0251602    304.0283511    306.065346    315.8582776    316.2454051
316.252388    356.0170616    379.8867687    379.9028164    389.4593553
389.465887    398.9098697    398.9201258    402.3218612    442.812214
442.8283538    459.1195053    463.6705827    483.2037523    489.3743431
493.601254    496.3183023    506.9665669    524.5004247    533.5068394
555.8532603    555.8732436    585.3820967    585.413541    596.5894156
596.6138994    609.6574518    609.6894488    609.9724499    653.3413886
653.4309884    673.5925014    699.8421478    703.111737    710.934934
711.1892528    712.3675864    714.7088193    722.9859913    723.0580321
736.7238024    777.0580727    818.7355944    826.4857469    830.7487811
830.7517077    835.264451    835.3255212    839.9551045    839.9929162
845.7006224    868.2538048    868.285788    913.6851676    913.7765665
925.5363839    968.0210321    969.4642975    970.7557419    972.2806178
972.8176581    992.0359219    995.6418939    1005.219678    1005.650874
1033.905694    1057.969007    1073.007889    1073.960601    1074.309456
1112.975085    1113.143267    1118.797909    1118.900251    1127.667024
1127.686265    1138.667307    1169.192273    1169.424683    1212.094942
1215.129525    1228.68681    1256.931606    1257.041552    1264.981973
1270.52156    1270.947497    1277.755253    1282.630421    1289.20724
1289.300608    1300.807871    1301.027352    1346.632463    1364.598557
1371.722915    1430.38453    1430.55596    1438.100893    1439.197533
1442.734664    1443.268783    1447.703906    1448.364404    1455.788051
1482.141873    1482.590711    1486.68772    1495.693135    1521.466288
1521.734291    1564.878892    1585.752362    1586.01102    1602.153523
1609.4264    1612.451356    1623.665601    1629.368955    1637.752806
1669.134077    1673.791353    1686.580087    1691.931144    1697.348844
1710.215714    1727.111308    1727.484063    1760.298465    1761.11658
1788.172512    1788.404972    1803.956162    1804.98358    1809.930864
1823.78967    1828.80486    1829.581067    1837.021072    1854.660103
1855.649574    1935.121515    1935.874245    1952.808776    1952.927515
1955.490348    1979.764982    1987.778708    1989.727161    1995.04969
1998.708381    2007.608495    2010.838773    2022.915334    2023.809711
2024.712487    2029.883776    2051.333891    2056.874594    2077.286448
2087.636428    2087.844281    2146.903208    2177.924753    2185.953341
2186.525251    2194.875851    2203.73406    2205.078399    2232.313303
2233.789239    2239.63656    2241.14649    2245.257004    2250.750583
2252.41489    2283.000484    2284.19441    2319.047085    2319.723838
2349.341449    2349.982103    2359.362313    2368.31407    2378.16499
2398.313715    2407.091317    2410.56839    2434.321861    2443.044002

```



2460.701273	2473.611355	2474.735925	2475.181206	2507.227532
2531.625372	2539.718552	2548.08828	2553.272881	2593.318105
2600.897702	2602.308568	2602.560716	2602.989274	2608.061746
2624.952997	2625.549818	2647.122508	2649.555144	2670.756855";

In[408]:=

```

evsFreeSquareβ05 = Flatten@ToExpression@StringSplit[strfr1];
evsFreeSquareβ4 = Flatten@ToExpression@StringSplit[strfr2];
evsFreeSquareβ101 = Flatten@ToExpression@StringSplit[strfr3];
bthisf1 = Bfree[2, 2 / 3, 1];
bthisf2 = Bfree[2, 1 / 5, 1];
bthisf3 = Bfree[2, 1 / 102, 1];

```

In[414]:=

```

bLiuthisf1 = -BDirLiu[2, -0.5, 1];
bLiuthisf2 = -BDirLiu[2, 3, 1];
bLiuthisf3 = -BDirLiu[2, 100, 1];

```

In[417]:=

```

xtsqf1 = Table[{x, MaTeX[x]}, {x, {750, 1500}}];
xtsqf2 = Table[{x, MaTeX[x]}, {x, {1000, 2000}}];
xtsqf3 = Table[{x, MaTeX[x]}, {x, {1300, 2600}}];
ytsqf = Table[{y, MaTeX["\ " <> ToString[y]]}, {y, 10, 40, 10}];

```

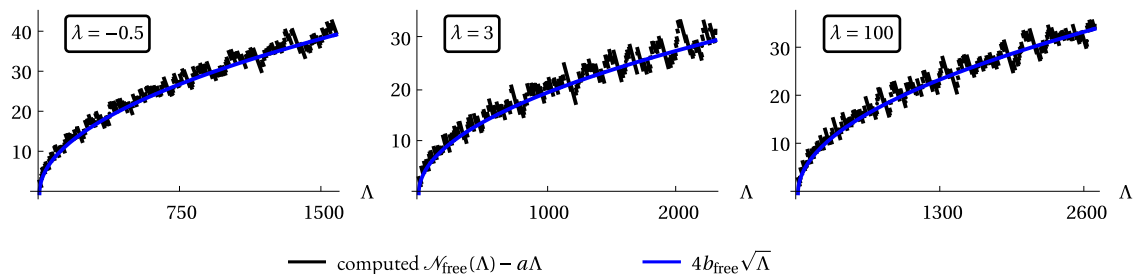
In[421]:=

```

figsqfree1 = Plot[{MyNlambdaSimple[evsFreeSquareβ05, Δ] - CWeyl[2, -0.5, 1] Δ,
  4 bthisf1 Sqrt[Δ]}, {Δ, 0, 1566}, PlotStyle → {Black, Blue},
  Ticks → {xtsqf1, ytsqf}, AxesLabel → {MaTeX["\\Lambda"], None}, Epilog →
  Inset[Framed[MaTeX["\\lambda=-0.5"], RoundingRadius → 2, ContentPadding →
    False, FrameMargins → Tiny], Scaled[{0.1, 1}], Scaled[{0, 1}]]];
figsqfree2 = Plot[
  {MyNlambdaSimple[evsFreeSquareβ4, Δ] - CWeyl[2, 3, 1] Δ, 4 bthisf2 Sqrt[Δ]},
  {Δ, 0, 2283}, PlotStyle → {Black, Blue}, Ticks → {xtsqf2, ytsqf},
  AxesLabel → {MaTeX["\\Lambda"], None}, Epilog → Inset[
  Framed[MaTeX["\\lambda=3"], RoundingRadius → 2, ContentPadding → False,
    FrameMargins → Tiny], Scaled[{0.1, 1}], Scaled[{0, 1}]]];
figsqfree3 = Plot[{MyNlambdaSimple[evsFreeSquareβ101, Δ] - CWeyl[2, 100, 1] Δ,
  4 bthisf3 Sqrt[Δ]}, {Δ, 0, 2671}, PlotStyle → {Black, Blue},
  Ticks → {xtsqf3, ytsqf}, AxesLabel → {MaTeX["\\Lambda"], None}, Epilog →
  Inset[Framed[MaTeX["\\lambda=100"], RoundingRadius → 2, ContentPadding →
    False, FrameMargins → Tiny], Scaled[{0.1, 1}], Scaled[{0, 1}]]];
legendsq = LineLegend[{Black, Blue, Red}, MaTeX[{"\\text{computed}
  }\\mathcal{N}_\\mathrm{free}}(\\Lambda) - a\\Lambda\\quad\\quad",
  "4b_\\mathrm{free}}\\sqrt{\\Lambda}\\quad\\quad"}],
  LegendFunction → None, LegendLayout → {"Row", 1}];
figsqfree = GraphicsColumn[{GraphicsRow[{figsqfree1, figsqfree2, figsqfree3},
  ImageSize → Full, Spacings → {Scaled[0.05], Automatic}],
  legendsq}, ImageSize → Full, Spacings → {0, Automatic}]

```

Out[425]=



In[\*]:= Export["figsqfree.pdf", figsqfree]

Out[\*]=

figsqfree.pdf

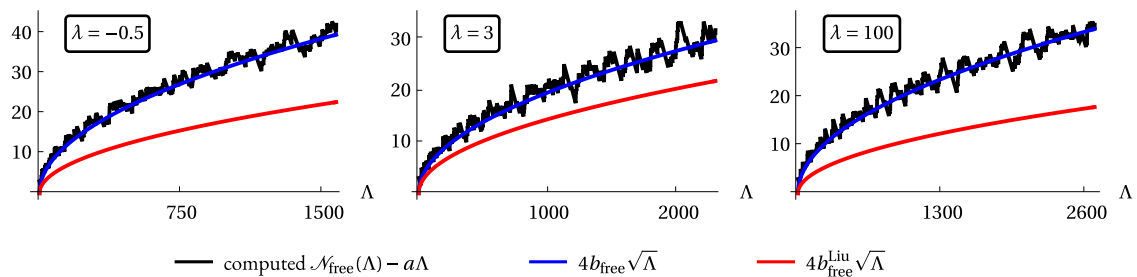
In[426]:=

```

figsqfree1new = Plot[{MyNlambdaSimple[evsFreeSquareβ05, Δ] - CWeyl[2, -0.5, 1] Δ,
  4 bthisf1 Sqrt[Δ], 4 bLiuthisf1 Sqrt[Δ]}, {Δ, 0, 1566},
  PlotStyle → {Black, Blue, Red}, Exclusions → None, Ticks → {xtsqf1, ytsqf},
  AxesLabel → {MaTeX["\\Lambda"], None}, Epilog → Inset[
    Framed[MaTeX["\\lambda=-0.5"], RoundingRadius → 2, ContentPadding → False,
      FrameMargins → Tiny], Scaled[{0.1, 1}], Scaled[{0, 1}]]];
figsqfree2new = Plot[{MyNlambdaSimple[evsFreeSquareβ4, Δ] - CWeyl[2, 3, 1] Δ,
  4 bthisf2 Sqrt[Δ], 4 bLiuthisf2 Sqrt[Δ]}, {Δ, 0, 2283},
  PlotStyle → {Black, Blue, Red}, Exclusions → None, Ticks → {xtsqf2, ytsqf},
  AxesLabel → {MaTeX["\\Lambda"], None}, Epilog → Inset[
    Framed[MaTeX["\\lambda=3"], RoundingRadius → 2, ContentPadding → False,
      FrameMargins → Tiny], Scaled[{0.1, 1}], Scaled[{0, 1}]]];
figsqfree3new = Plot[{MyNlambdaSimple[evsFreeSquareβ101, Δ] - CWeyl[2, 100, 1] Δ,
  4 bthisf3 Sqrt[Δ], 4 bLiuthisf3 Sqrt[Δ]}, {Δ, 0, 2671},
  PlotStyle → {Black, Blue, Red}, Exclusions → None, Ticks → {xtsqf3, ytsqf},
  AxesLabel → {MaTeX["\\Lambda"], None}, Epilog → Inset[
    Framed[MaTeX["\\lambda=100"], RoundingRadius → 2, ContentPadding → False,
      FrameMargins → Tiny], Scaled[{0.1, 1}], Scaled[{0, 1}]]];
legendsqnew = LineLegend[{Black, Blue, Red}, MaTeX[{"\\text{computed}
  }\\mathcal{N}_{\\mathrm{free}}(\\Lambda) - a\\Lambda\\quad\\quad",
  "4b_{\\mathrm{free}}\\sqrt{\\Lambda}\\quad\\quad",
  "4b_{\\mathrm{free}}^{\\mathrm{Liu}}\\sqrt{\\Lambda}\\quad\\quad"}],
  LegendFunction → None, LegendLayout → {"Row", 1}];
figsqfreenew =
  GraphicsColumn[{GraphicsRow[{figsqfree1new, figsqfree2new, figsqfree3new},
    ImageSize → Full, Spacings → {Scaled[0.05], Automatic}],
  legendsqnew}, ImageSize → Full, Spacings → {0, Automatic}]

```

Out[430]=



In[\*]:= Export["figsqfreenew.pdf", figsqfreenew]

Out[\*]=

figsqfreenew.pdf